

Kurs wyrównawczy

28.10.2021

Lista 1.

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$$\begin{aligned} a^m \cdot a^n &= a^{m+n} \\ (a^m)^n &= a^{m \cdot n} \\ \frac{a^m}{a^n} &= a^{m-n} \\ \frac{a^m}{a^n} &= a^{m-n} \end{aligned} \quad a^{-1} = \frac{1}{a}$$

$$(a \cdot b)^n = a^n \cdot b^n$$

1. Oblicz

$$a) \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$$b) (0,1)^{-4} = \left(\frac{1}{10}\right)^{-4} = 10000$$

$$c) 2^{-2} \cdot \left(\frac{1}{4}\right)^{-2} = \left(\frac{1}{2}\right)^2 \cdot 4^2 = \frac{1}{4} \cdot 16 = 4$$

$$g) \left(1\frac{1}{6}\right)^{-3} \cdot \left(\frac{2}{7}\right)^{-2} = \left(\frac{7}{6}\right)^{-3} \cdot \left(\frac{2}{7}\right)^{-2} = \left(\frac{6}{7}\right)^3 \cdot \left(\frac{7}{2}\right)^2 = \frac{6 \cdot 6 \cdot 6}{7 \cdot 7 \cdot 7} \cdot \frac{49}{4} = \frac{6 \cdot 9}{7} = \frac{54}{7}$$

$$h) (2,8)^{-3} \cdot \left(\frac{5}{14}\right)^{-4} = \left(\frac{10}{28}\right)^{-3} \cdot \left(\frac{14}{5}\right)^4 = \frac{(5 \cdot 2)^3}{(2 \cdot 14)^3} \cdot \frac{14^4}{5^4} = \frac{5^3 \cdot 2^3}{2^3 \cdot 14^3} \cdot \frac{14^4}{5^4} = \frac{14}{5}$$

$$j) \frac{(3^{-2})^{-3}}{3^8} = \dots = \frac{1}{9}$$

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

zad. 2. Doprowadź do najprostszej postaci:

$$a) \sqrt{8} - \sqrt{32} + \sqrt{128} = \sqrt{8} - \sqrt{4 \cdot 8} + \sqrt{4 \cdot 4 \cdot 8} = \sqrt{8} - 2\sqrt{8} + 4\sqrt{8} = 3\sqrt{8} = 6\sqrt{2}$$

$$b) \sqrt{27} - \sqrt{48} + \sqrt{75} = \sqrt{3 \cdot 9} - \sqrt{3 \cdot 16} + \sqrt{3 \cdot 25} = 3\sqrt{3} - 4\sqrt{3} + 5\sqrt{3} = 4\sqrt{3}$$

$$c) (\sqrt{3} + 2\sqrt{2}) \cdot (3\sqrt{2} - 4\sqrt{3}) = 3\sqrt{6} - 4\sqrt{9} + 6\sqrt{4} - 8\sqrt{6} = -5\sqrt{6} - 12 + 12 = -5\sqrt{6}$$

$$a^n \cdot a^m = a^{n+m}$$

$$(a^n)^m = a^{n \cdot m}$$

Zad. 4.

$$a) \left(\frac{3}{8}\right)^{\frac{8}{3}} \cdot \left(\frac{8}{3}\right)^{\frac{3}{8}} = \left(\frac{3}{8}\right)^{\frac{8}{3}} \cdot \left(\left(\frac{3}{8}\right)^{-1}\right)^{\frac{3}{8}} = \left(\frac{3}{8}\right)^{\frac{8}{3}} \cdot \left(\frac{3}{8}\right)^{-\frac{3}{8}} = \left(\frac{3}{8}\right)^{\frac{8}{3} - \frac{3}{8}} = \left(\frac{3}{8}\right)^{\frac{55}{24}}$$

$$\frac{8}{3} - \frac{3}{8} = \frac{64 - 9}{24} = \frac{55}{24}$$

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Zad. 4c

$$0,75^{-3} \cdot \sqrt[3]{\frac{10}{27}} = \left(\frac{3}{4}\right)^{-3} \cdot \sqrt[3]{\frac{64}{27}} = \left(\frac{4}{3}\right)^3 \cdot \sqrt[3]{\frac{4^3}{3^3}}$$
$$= \left(\frac{4}{3}\right)^3 \cdot \frac{(4^3)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}} = \left(\frac{4}{3}\right)^3 \cdot \frac{4}{3} = \frac{4^4}{3^4} = \frac{256}{81}$$

$$64 = 8^2 = 4^3$$

$$\sqrt[k]{a^k} = a^{\frac{k}{k}} = a^1$$

$$\sqrt[k]{a} = a^{\frac{1}{k}}$$

Zad. 5. Usun' niewymierność z mianownika :

$$a) \frac{6}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{6 \cdot (3-\sqrt{3})}{9-3} = \frac{6(3-\sqrt{3})}{6} = 3-\sqrt{3}$$

$$b) \frac{6}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{6(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{6\sqrt{3}+6\sqrt{2}}{3-2} = 6\sqrt{3}+6\sqrt{2}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$\begin{aligned}
 \left[\left(-\frac{2}{3}\right)^{-3} + 3 \cdot 2^{-3} \right]^{-2} &= \left[\left(-\frac{3}{2}\right)^3 + 3 \cdot 2^3 \right]^{-2} = \left(-\frac{27}{8} + 3 \cdot 2^3 \right)^{-2} \\
 &= \left(-\frac{27}{8} + 24 \right)^{-2} = \left(\frac{-27+24 \cdot 8}{8} \right)^{-2} = \\
 &= \left(\frac{8}{-27+24 \cdot 8} \right)^2 = \left(\frac{8}{3(-9)+3 \cdot 8 \cdot 8} \right)^2 = \\
 &= \left(\frac{8}{3(-9+64)} \right)^2 = \left(\frac{8}{3 \cdot 55} \right)^2 = \left(\frac{8}{165} \right)^2
 \end{aligned}$$

$$\begin{aligned}
 \frac{(0,1)^{-1} - (0,4)^0}{2^{\frac{2}{3}} \cdot \left(\frac{2}{3}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} &= \\
 = \frac{10-1}{\frac{8}{3} \cdot \left(\frac{3}{2}\right)^3 + (-3)} &= \frac{9}{\frac{8}{2} \cdot \frac{27}{8} - 3} = \frac{9}{9-3} = \frac{9}{6} = \frac{3}{2}
 \end{aligned}$$

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$$(a^m)^n = a^{m \cdot n}$$

$$\begin{aligned} & \left(\frac{9}{16}\right)^{\frac{1}{10}} \cdot \left(\frac{25}{36}\right)^{-\frac{3}{2}} - \left[\left(\frac{4}{3}\right)^{-\frac{1}{2}}\right]^{-\frac{2}{5}} \cdot \left(\frac{6}{5}\right)^{-3} = \\ & = \left(\frac{16}{9}\right)^{\frac{1}{10}} \cdot \left(\frac{25}{36}\right)^{-\frac{3}{2}} - \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 = \\ & = \left(\frac{16}{9}\right)^{\frac{1}{10}} \cdot \left(\frac{26}{25}\right)^{\frac{3}{2}} - \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 = \\ & = \left(\left(\frac{16}{9}\right)^{\frac{1}{2}}\right)^{\frac{1}{5}} \cdot \left(\frac{25}{36}\right)^{\frac{3}{2}} - \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 = \\ & = \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{25}{36}\right)^{\frac{1}{2}} \cdot \left(\frac{25}{36}\right)^{\frac{3}{2}} - \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 = \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 - \left(\frac{4}{3}\right)^{\frac{1}{5}} \cdot \left(\frac{5}{6}\right)^3 = 0 \end{aligned}$$

$$\frac{1}{10} = \frac{1}{2} \cdot \frac{1}{5}$$

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