On the density of truth in Dummett's logic (Extended Abstract) *

Zofia Kostrzycka[1], Marek Zaionc[2]

[1] Politechnika Opolska
Luboszycka 3, 45-036 Opole, Poland
E-mail zkostrz@polo.po.opole.pl

[2] Computer Science Department, Jagiellonian University,
Nawojki 11, 30-072 Krak"ow, Poland.
E-mail zaionc@ii.uj.edu.pl

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Abstract

For the given logical calculus we investigate the size of the fraction of true formulas of a certain length n against the number of all formulas of such length. We are especially interested in asymptotic behaviour of this fraction when n tends to infinity. If the limit of the fraction exists it represents a number which we may call *the density of truth* for the investigated logic. In this paper we apply this approach to the Dummett intermediate linear logic (see [?]). Actually, this paper shows the exact density of this logic and demonstrates that it covers a substantial part of classical propositional calculus. Despite using strictly mathematical means to solve all discussed problems, this paper in fact, may have a philosophical impact on understanding how much the phenomenon of truth is sporadic or frequent in random mathematics sentences.

1 Introduction

The research described in this paper is a part of a project of quantitative investigations in logic. We investigate the language $\mathcal{F}^{\{\rightarrow,\neg\}}$ consisting of implicational-negational formulas over one propositional variable. For some subclass $A \subset \mathcal{F}^{\{\rightarrow,\neg\}}$ we may associate the density $\mu(A)$ as:

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$$\mu(A) = \lim_{n\to\infty} \frac{\#\{t \in A : \|t\| = n\}}{\#\{t \in \mathcal{F}^{\{\to,\neg\}} : \|t\| = n\}} \tag{1}$$

where $\|.\|$ stands for the length of formula defined in the conventional way. The number $\mu(A)$ if exists, is an asymptotic probability of finding a formula from the class A among all formulas from $\mathcal{F}^{\{\to,\neg\}}$ and the asymptotic density of the set A in the set $\mathcal{F}^{\{\to,\neg\}}$ as well.

The paper is a natural continuation of the problem concerning the *density of truth* in classical logic of one variable. The result published in [**?**] proved the existence of the *density of truth* for classical (and intuitionistic) logic of implication of one variable. In the paper [**?**] it is shown that the *density* also exists for the implicational-negational formulas of one variable. In this note we prove the similar result for Dummett's intermediate linear logic LC.

2 Implicational - negational formulas

The language of implicational - negational formulas of one propositional variable a consists of formulas $\mathcal{F}^{\{\to,\neg\}}$ built from a by means of negation and implication only.

$$\begin{array}{rcl} a & \in & \mathcal{F}^{\{\to,\neg\}} \\ \phi \to \psi \in \mathcal{F}^{\{\to,\neg\}} & iff & \phi \in \mathcal{F}^{\{\to,\neg\}} \text{ and } \psi \in \mathcal{F}^{\{\to,\neg\}} \\ \neg\phi \in \mathcal{F}^{\{\to,\neg\}} & iff & \phi \in \mathcal{F}^{\{\to,\neg\}} \end{array}$$

We start this section by defining the Dummett logic LC semantically (see [**?**]).

Definition 1 *By Dummett's matrix we mean the infinite-valued characteristic matrix $M_\omega = \langle |M_\omega|, \sim, \Rightarrow, \{1\}\rangle$, where the set $|M_\omega| = \mathbb{N} \cup \{\omega\}$ is equipped with two operations $\{\sim, \Rightarrow\}$ defined as:*

$$\sim p = \begin{cases} \omega & if \quad p < \omega \\ 1 & if \quad p = \omega \end{cases} \qquad p \Rightarrow q = \begin{cases} 1 & if \quad p \geq q \\ q & if \quad p < q \end{cases}$$

Definition 2 *By the valuation of our language $\mathcal{F}^{\{\to,\neg\}}$ in the matrix $|M_\omega|$ we mean any function $v : \mathcal{F}^{\{\to,\neg\}} \to |M_\omega|$ satisfying $v(\phi \to \psi) = v(\phi) \Rightarrow v(\psi)$ and $v(\neg\phi) = \sim v(\phi)$. A formula α is a tautology iff $v(\alpha) = 1$ for every valuation $v : \mathcal{F}^{\{\to,\neg\}} \to |M_\omega|$. By $E(M_\omega)$ we mean the set of all tautologies in LC.*

First, we divide the set of all formulas into several classes according to the behaviour of each formula on all possible evaluations. Since we have formulas built with exactly one propositional variable a we can enumerate valuations by the elements of $|M_\omega|$ as follows

$$v_i(a) \quad = \quad i \quad \text{for all } i \in |M_\omega| \tag{2}$$

By the sequence of valuations α we mean any function $\alpha : |M_\omega| \to |M_\omega|$. Sequences of valuations are ordered componentwise by $\alpha \leq \beta$ iff for all $i \in |M_\omega|$ $\alpha(i) \leq \beta(i)$ and form a poset. On sequences we may introduce operations $\{\sim, \Rightarrow\}$ also componentwise by: $\alpha \Rightarrow \beta = \gamma$ if $\gamma(i) = \alpha(i) \Rightarrow \beta(i)$ and $\sim \alpha = \beta$ if $\beta(i) =\sim \alpha(i)$. Each sequence of valuations α defines uniquely the set of formulas $F^\alpha \subset \mathcal{F}^{\{\to,\neg\}}$ which are undistinguishable by all valuations. Let

$$F^\alpha \quad = \quad \Big\{\phi \in \mathcal{F}^{\{\to,\neg\}} : \forall i \in |M_\omega| \quad v_i(\phi) = \alpha(i)\Big\} \tag{3}$$

For example, the formula a belongs to the class F^α for the sequence $\alpha(i) = i, \quad \forall i \in |M_\omega|$, while the formula $\neg a \to a$ lays in the class F^α for the sequence $\alpha(i) = 1, \quad \forall i \in \mathbb{N}$ and $\alpha(\omega) = \omega$. It is obvious that classes are disjoint so $F^\alpha \cap F^\beta = \emptyset$ for $\alpha \neq \underline{=}eta$ and $\bigcup_\alpha F^\alpha = \mathcal{F}^{\{\to,\neg\}}$. The sequence α is called nonempty if the set of formulas $F^\alpha \neq \emptyset$. Our first task is to separate all nonempty sequences of valuations. We can easily see that the class F^α for the sequence $\alpha(i) = i, \quad \forall i \in |M_\omega|$ is nonempty since our initial formula a lays in F^α. Closing the set of nonempty sequences of valuations by operations $\{\sim, \Rightarrow\}$ we isolate exactly six sequences. Bellow we list all six classes together with appropriate sequences. In order to simplify notations we are going to call the classes A, B, C, D, E, G.

$$\begin{array}{lcll}
A & = & F^{\alpha_A} & \alpha_A(i) = \omega \\
B & = & F^{\alpha_B} & \alpha_B(i) = i \\
C & = & F^{\alpha_C} & \alpha_C(i) = \omega \text{ for } i < \omega \text{ and } \alpha_C(\omega) = 1 \\
D & = & F^{\alpha_D} & \alpha_D(i) = i \text{ for } i < \omega \text{ and } \alpha_D(\omega) = 1 \\
E & = & F^{\alpha_E} & \alpha_E(i) = 1 \text{ for } i < \omega \text{ and } \alpha_E(\omega) = \omega \\
G & = & F^{\alpha_G} & \alpha_G(i) = 1
\end{array}$$

As we can see the class G establishes the set $E(M_\omega)$ of all tautologies in LC. Semantic operations $\{\sim, \Rightarrow\}$ on these classes defined by $F^\alpha \Rightarrow F^\beta = F^{\alpha \Rightarrow \beta}$ and $\sim F^\alpha = F^{\sim\alpha}$ can be displayed by the following truth table:

$\Rightarrow$	A	B	C	D	E	G	$\sim$
A	G	G	G	G	G	G	G
B	C	G	C	G	G	G	C
C	E	E	G	G	E	G	E
D	A	E	C	G	E	G	A
E	C	D	C	D	G	G	C
G	A	B	C	D	E	G	A

Table 1.

The order on classes F^α is defined as $F^\alpha \leq F^{\alpha'}$ iff $\alpha \geq \alpha'$. It forms the following lattice diagram with the class of tautologies G being on the top:

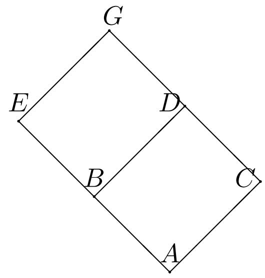

Diagram 1.

For technical reasons we are also going to consider two posets obtained from the one above by appropriate identification. The first one is a three elements chain obtained by identifying classes E and G, B and D as well as A and C. We will name such classes as EG, BD and AC. The second one, four elements Boolean algebra, is obtained by identifying classes D and G, and B and E. Accordingly we will call such classes DG and BE. They have following diagrams:

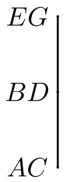

Diagram 2.

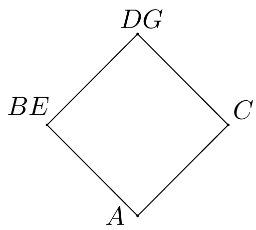

Diagram 3.

The operations $\{\sim, \Rightarrow\}$ on new classes in new posets are given by the following truth tables:

$\Rightarrow$	AC	BD	EG	$\sim$
AC	EG	EG	EG	EG
BD	AC	EG	EG	AC
EG	AC	BD	EG	AC

Table 2.

$\Rightarrow$	A	C	BE	DG	$\sim$
A	DG	DG	DG	DG	DG
C	BE	DG	BE	DG	BE
BE	C	C	DG	DG	C
DG	A	C	BE	DG	A

Table 3.

As we can observe, the first truth table describes operations in the Gödel 3 valued matrix, while the second one is a matrix of all valuations associated with the standard classical logic of one variable.

3 Counting Formulas

In this section we present some properties of numbers characterizing the amount of formulas in different classes defined in our language. First, let us establish the way of measuring the length of formulas. By $\|\phi\|$ we mean the length of formula ϕ which is the total number of characters in the formula, including implication and negation signs. Brackets, which are sometimes necessary, are not included in the length of formula. Formally:

$$\begin{aligned} \|a\| &= 1 \\ \|\phi \to \psi\| &= \|\phi\| + \|\psi\| + 1 \\ \|\neg\phi\| &= \|\phi\| + 1. \end{aligned}$$

Definition 3 *By* $\mathcal{F}_n^{\{\to,\neg\}}$ *we mean the set of formulas of the length* $n-1$*. Subclasses* $A_n, B_n, C_n, D_n, E_n, G_n$ *and additional subclasses* EG_n*,* BD_n*,* AC_n*,* DG_n*,* BE_n *of formulas of the length* $n-1$ *are defined accordingly by:*

$$\begin{aligned} A_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap A & B_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap B \\ C_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap C & D_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap D \\ E_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap E & G_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap G \\ EG_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap EG & BD_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap BD \\ AC_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap AC & DG_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap DG \\ BE_n &= \mathcal{F}_n^{\{\to,\neg\}} \cap BE & & \end{aligned}$$

We can see that for any $n \in \mathbb{N}$ *the number of formulas in* $\mathcal{F}_n^{\{\to,\neg\}}$ *is finite and will be denoted as* $\left|\mathcal{F}_n^{\{\to,\neg\}}\right|$*. Consequently all subclasses listed above are also finite for all* $n \in \mathbb{N}$*.*

4 Generating functions

The main tool we use for dealing with asymptotics of sequences of numbers are *generating functions.* A nice exposition of the method can be found in [**?**] and [**?**]. Our main task in this paper is to determine limits of various sequences of real numbers. For this purpose combinatorics has developed an extremely powerful tool in the form of generating series and generating functions. Let $A = (A_0, A_1, A_2, \ldots)$ be a sequence of real numbers. The *ordinary generating series* for A is the formal power series $\sum_{n=0}^{\infty} A_n z^n$ and, of course, the formal power series is in one-to-one correspondence to the sequence. However, considering z as a complex variable, this series, as it is known from the theory of analytic functions, converges uniformly to a function $f_A(z)$ in some open disc $\{z \in \mathcal{C} : |z| < R\}$ of maximal diameter, and $R \geq 0$ is called its radius of convergence. So with the sequence A we can associate a complex function $f_A(z)$, called the *ordinary generating function* for A, defined in a neighborhood of 0.

This correspondence is one-to-one again (unless $R = 0$), since the expansion of a complex function $f(z)$, analytic in a neighborhood of z_0, into a power series $\sum_{n=0}^{\infty} A_n(z - z_0)^n$ is unique, and moreover, this series is the Taylor series, given by

$$A_n = \frac{1}{n!}\frac{d^n f}{dz^n}(z_0). \tag{4}$$

Many questions concerning the asymptotic behaviour of A can be efficiently resolved by analyzing the behaviour of f_A at the complex circle $|z| = R$.
This is the approach we take to determine the asymptotic fraction of tautologies and many other classes of formulas among all formulas of a given length.
The key tool will be the following result due to Szegö [**?**] [Thm. 8.4]; see also [**?**] [Thm. 5.3.2], which relates to the generating functions of numerical sequences with limit of the fractions being investigated. For the technique of proof described below please consult also [**?**] as well as [**?**]. We need the following much simpler version of the Szegö lemma.

Lemma 4 *Let $v(z)$ be analytic in $|z| < 1$ with $z = 1$ the only singularity at the circle $|z| = 1$. If $v(z)$ in the vicinity of $z = 1$ has an expansion of the form*

$$v(z) = \sum_{p \geq 0} v_p (1 - z)^{\frac{p}{2}}, \tag{5}$$

where $p > 0$, and the branch chosen above for the expansion equals $v(0)$ for $z = 0$, then

$$[z^n]\{v(z)\} = v_1 \binom{1/2}{n}(-1)^n + O(n^{-2}). \tag{6}$$

The symbol $[z^n]\{v(z)\}$ stands for the coefficient of z^n in the exponential series expansion of $v(z)$.

5 Calculating generating functions

In this section we are going to find the generating function for the class of tautologies G. First, recall the following two generating functions calculated in [**?**] for sequences $\left|\mathcal{F}_n^{\{\rightarrow,\neg\}}\right|$ and $|DG_n|$.

$$\begin{aligned} f_F(z) &= \frac{1-z}{2} - \frac{\sqrt{(z+1)(1-3z)}}{2} \\ f_{DG}(z) &= \frac{1}{8}\Big(8 - \sqrt{2}\sqrt{1+6z-z^2-Y} - \sqrt{2}\sqrt{1+6z+7z^2-Y} - \\ &\quad 2\sqrt{1-10z+3z^2-Y+\sqrt{1+6z-z^2-Y}\sqrt{1+6z+7z^2-Y}}\Big), \\ &\quad \text{where } Y = (1-z)\sqrt{(1+z)(1-3z)} \end{aligned} \tag{7}$$

Next, let us compute the generating function for the class EG_n.

Lemma 5 *The numbers $|AC_n|$, $|BD_n|$ and $|EG_n|$ are given by the recursions:*

$$\begin{aligned}
|AC_0| &= |AC_1| = |AC_2| = 0, \quad |AC_3| = 1 \\
|AC_n| &= |BD_{n-1}| + |EG_{n-1}| + \sum_{i=1}^{n-2}(|BD_i| + |EG_i|)|AC_=n-i| \qquad (8) \\
|BD_0| &= |BD_1| = 0, \quad |BD_2| = 1 \\
|BD_n| &= \sum_{i=1}^{n-2} |EG_i||BD_{n-i}| \qquad (9) \\
|EG_0| &= |EG_1| = |EG_2| = |EG_3| = 0, \ |EG_4| = 2 \\
|EG_n| &= |F_n| - (|AC_n| + |BD_n|). \qquad (10)
\end{aligned}$$

Proof. It follows easily from Table 2. Formulas from class AC can be obtained as negations of formulas from classes BD and EG. This part is responsible for the component $|BD_{n-1}| + |EG_{n-1}|$. Analyzing Table 2 we also can notice that the amount of formulas from AC in the form of implications depends only on the same classes BD and EG and these from AC. This fact is described in the fragment $\sum_{i=1}^{n-2}(|BD_i| + |EG_i|)|AC_{n-i}|$. So, equality (**??**) has been proven. The formulas in the class BD can only be implications of formulas from classes EG and BD. This gives (**??**). The last equation (**??**) is obvious.

Lemma 6 *The generating function f_{BD} for sequence of numbers $|BD_n|$ is:*

$$\begin{aligned}
f_{BD}(z) &= \frac{1}{8}\Bigg(2^{3/4}\sqrt{(X - z + 1)\sqrt{Y - z^2 + 6z + 1} + \sqrt{2}(Y + 15z^2 + 2z + 1)} - \\
&\quad \sqrt{2}\sqrt{Y - z^2 + 6z + 1} - X + z - 1\Bigg), \qquad (11) \\
&\quad \textit{where } X = \sqrt{(z+1)(1-3z)} \quad \textit{and } Y = (1-z)X.
\end{aligned}$$

Proof. First, we can observe the generating functions f_{AC}, f_{BD} i f_{EG} for numbers $|AC_n|$, $|BD_n|$ and $|EG_n|$ satisfy the following equalities:

$$\begin{aligned}
f_{AC} &= (f_{BD} + f_{EG})z + (f_{BD} + f_{EG})f_{AC} \qquad (12) \\
f_{BD} &= f_{EG}f_{BD} + z^2 \qquad (13) \\
f_{EG} &= f_F - (f_{AC} + f_{BD}). \qquad (14)
\end{aligned}$$

The recurrence (**??**) corresponds to multiplication of power series and then gives the equality (**??**). The quadratic term z^2 in (**??**) corresponds to the first non-zero coefficient in the power series of f_{BD}. In the same manner we can see that the fragment $\sum_{i=1}^{n-2}(|BD_i| + |EG_i|)|AC_{n-i}|$ corresponds to the multiplication $(f_{BD}+f_{EG})f_{AC}$, while the term $|BD_{n-1}|+|EG_{n-1}|$ corresponds to the function $(f_{BD} + f_{EG})z$. Solving the system of equations (**??**), (**??**) and (**??**) we obtain (**??**). Note, that we choose the solution satisfying the boundary conditions $f_{BD}(0) = 0$. Now, we can attack the problem of finding the generating function for class of tautologies G.

It is easy to observe equation (**??**) from Table 1 and equations (**??**)-(**??**) from Diagrams 1-3.

$$
\begin{aligned}
f_B &= f_G f_B + z^2 && (15)\\
f_G &= f_{DG} - f_D && (16)\\
f_D &= f_{BD} - f_B && (17)\\
f_G &= f_{DG} - f_{BD} + f_B && (18)
\end{aligned}
$$

From (**??**) and (**??**) we get

$$f_B = (f_{DG} - f_{BD} + f_B) f_B + z^2. \qquad (19)$$

By solving (**??**) with boundary condition $f_B(0) = 0$ we get function f_B which is presented here in terms of already known functions f_{DG} and f_{BD} see (**??**) and (**??**):

$$f_B = \frac{1}{2}(1 - f_{DG} + f_{BD} - \sqrt{(f_{DG} - f_{BD} - 1)^2 - 4z^2}). \qquad (20)$$

Now, from (**??**) and (**??**) we calculate the function f_G. For simplicity, we can present it again in terms of functions f_{DG} and f_{BD}.

Lemma 7 *Generating function for the sequence of tautologies is:*

$$f_G = \frac{1}{2}(f_{DG} - f_{BD} + 1 - \sqrt{(f_{DG} - f_{BD} - 1)^2 - 4z^2}). \qquad (21)$$

6 From generating functions to asymptotic densities

To apply the Szegö lemma we have to have functions which are analytic in the open disc $|z| < 1$, and the nearest singularity is at $z_0 = 1$. For that purpose we are going to calibrate functions f_F and f_G in the following way:

$$
\begin{aligned}
\widehat{f_F}(z) &= f_F\left(\tfrac{z}{3}\right) & \widehat{f_{DG}}(z) &= f_{DG}\left(\tfrac{z}{3}\right)\\
\widehat{f_{BD}}(z) &= f_{BD}\left(\tfrac{z}{3}\right) & \widehat{f_G}(z) &= f_G\left(\tfrac{z}{3}\right).
\end{aligned}
$$

After appropriate simplification of the above expressions we get the following:

$$
\begin{aligned}
\widehat{f_F}(z) &= \frac{1}{6}\left(3 - z - \sqrt{3}\sqrt{(z+3)(1-z)}\right) && (22)\\
\widehat{f_{DG}}(z) &= \frac{1}{24}\Big(24 - \sqrt{2}\sqrt{9 + 18z - z^2 - Y} - \sqrt{2}\sqrt{9 + 18z + 7z^2 - Y} - \\
&\quad 2\sqrt{9 - 30z + z^2 - Y + \sqrt{9 + 18z - z^2 - Y}\sqrt{9 + 18z + 7z^2 - Y}}\Big) && (23)
\end{aligned}
$$

$$\widehat{f_{BD}}(z) = \frac{1}{24}\Big(z - 3 - \sqrt{3}X - \sqrt{2}\sqrt{9 + 18z - z^2 - Y} + 2^{3/4}\sqrt{(\sqrt{3}X - z + 3)\sqrt{9 + 18z - z^2 - Y} + \sqrt{2}(15z^2 + 6z + 9 - Y)}\Big) \quad (24)$$

$$\widehat{f_G}(z) = \frac{1}{2}(\widehat{f_{DG}}(z) - \widehat{f_{BD}}(z) + 1 = -\sqrt{(\widehat{f_{DG}}(z) - \widehat{f_{BD}}(z) - 1)^2 - \frac{4}{9}z^2}), \quad (25)$$

where $X = \sqrt{(z+3)(1-z)}$ and $Y = \sqrt{3}(= z - 3)X$.

Note that relations between power series' of appropriate functions are such as $[z^n]\{f(z)\} = \Big([z^n]\{\widehat{f}(z)\}\Big)\, 3^n$.

Lemma 8 *$z_0 = 1$ is the only singularity of $\widehat{f_F}$ and $\widehat{f_G}$ located in $|z| \le 1$.*

Proof. It is easy to observe the function $\widehat{f_F}(z)$ has only singularities at $z = 1$ and $z = -3$. To make sure the function $\widehat{f_G}(z)$ has the nearest one at $z = 1$, we had to solve the following complicated equations:

$$9 + 18z - z^2 - Y = 0 \quad (26)$$
$$9 + 18z + 7z^2 - Y = 0 \quad (27)$$
$$9 - 30z + z^2 - Y + \sqrt{9 + 18z - z^2 - Y}\sqrt{9 + 18z + 7z^2 - Y} = 0 \quad (28)$$
$$(\sqrt{3}X - z + 3)\sqrt{9 + 18z - z^2 - Y} + \sqrt{2}(15z^2 + 6z + 9 - Y) = 0 \quad (29)$$
$$(\widehat{f_{DG}}(z) - \widehat{f_{BD}}(z) - 1)^2 - \frac{4}{9}z^2 = 0 \quad (30)$$

where $X = \sqrt{(z+3)(1-z)}$ and $Y = \sqrt{3}(z- = 3)X$.

To do that we had to extensively used *the Mathematica* package and it occurred that all solutions which are different from $z = 1$ are situated outside the disc $|z| \le 1$.

Theorem 9 *Expansions of functions $\widehat{f_F}$ and $\widehat{f_B}$ in a neighborhood of $z = 1$ are as follows:*

$$\widehat{f_F}(z) = f_0 + f_1\sqrt{1-z} + ...$$
$$\widehat{f_G}(z) = g_0 + g_1\sqrt{1-z} + ...$$

where

$$f_0 = \frac{1}{3}, \quad f_1 = -\frac{1}{\sqrt{3}}, \quad g_0 = 0.094\ldots, \quad g_1 = -0.228172\ldots$$

Proof. The coefficients f_0 and f_1 have been found in [**?**] using *the Mathematica* package. Similarly we found g_0 i g_1. Now, we can calculate the density of implicational-negational part of Dummett's linear logic of one variable. By applying Szegö lemma we get as follows.

Theorem 10

$$\mu(G) = \lim_{n\to\infty} \frac{|G_n|}{|\mathcal{F}_n^{\{\to,\neg\}}|} = \lim_{n\to\infty} \frac{(g_1\binom{1/2}{n}(-1)^n + O(n^{-2}))3^n}{(f_1\binom{1/2}{n}(-1)^n + O(n^{-2}))3^n}$$
$$= \lim_{n\to\infty} \frac{g_1}{f_1}(1+o(1)) = \frac{g_1}{f_1} \approx 0.395205$$

As we know Dummett's logic is a proper subset of classical one. Finally, the result above can be employed to calculate the size of fragment of Dummett's logic inside classical logic. The density of the implicational-negational part of the classical logic of one variable is about 0.4232 (see [**?**]). So, the probability of finding a linear tautology among classical ones is the following.

Theorem 11 *The relative probability of finding a linear tautology among classical ones is more then 93 %.*

Proof. Class DG in Diagram 3 is in fact the class of classical tautologies (see [**?**]). We already know asymptotics $\lim_{n\to\infty} \frac{|G_n|}{|\mathcal{F}_n^{\{\to,\neg\}}|}$ and $\lim_{n\to\infty} \frac{|DG_n|}{|\mathcal{F}_n^{\{\to,\neg\}}|}$ therefore

$$\lim_{n\to\infty} \frac{|G_n|}{|DG_n|} = \frac{\lim_{n\to\infty} \frac{|G_n|}{|\mathcal{F}_n^{\{\to,\neg\}}|}}{\lim_{n\to\infty} \frac{|DG_n|}{|\mathcal{F}_n^{\{\to,\neg\}}|}} = \frac{0.395305\ldots}{0.44232\ldots} \approx 93\%.$$

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